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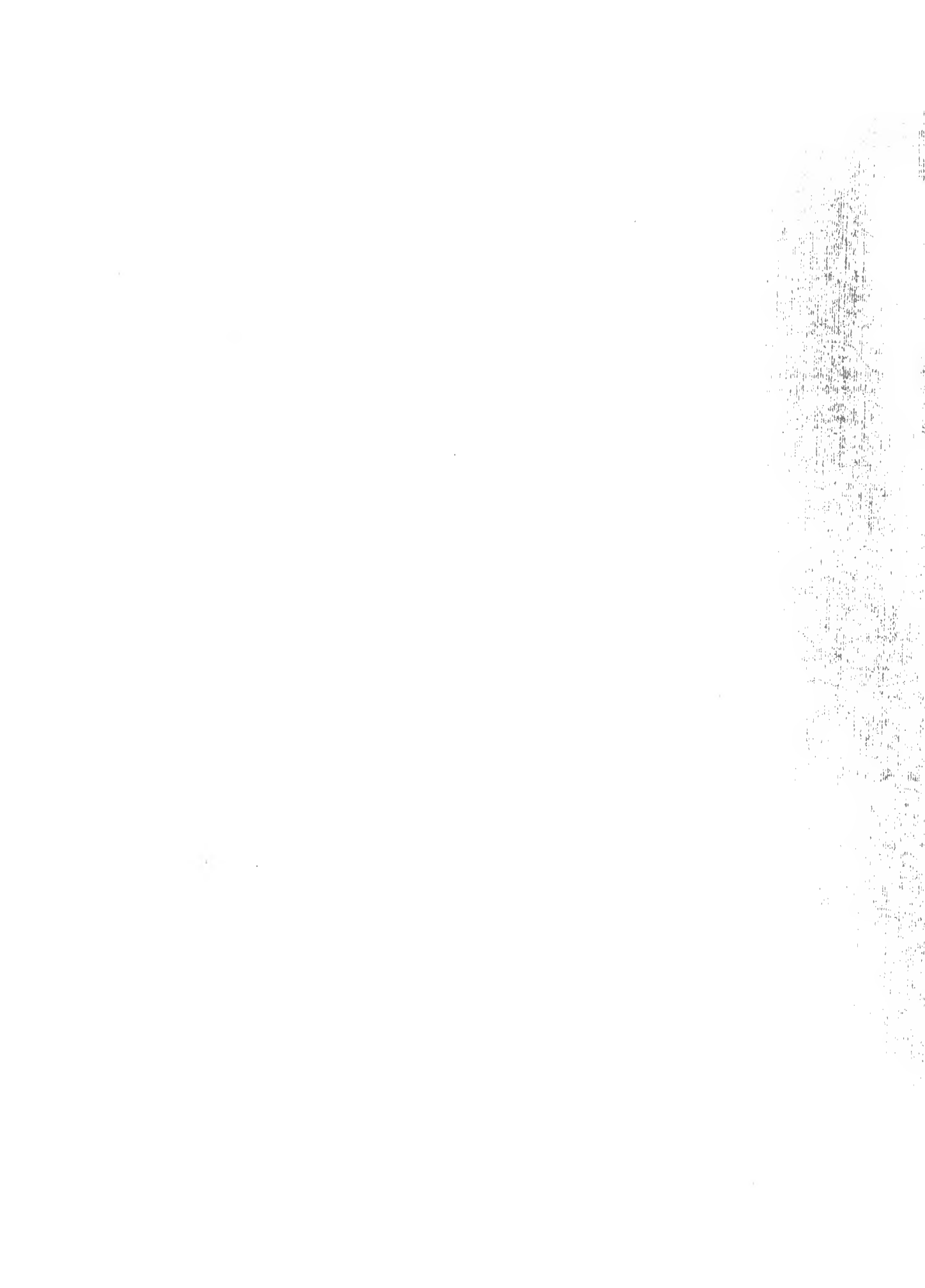
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Pareto Optimality and Bidding for Contracts

Susan I. Cohen

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Pareto Optimality and Bidding for Contracts

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ABSTRACT

In the "standard" principal-agent model in which there is one principal contracting with one risk-neutral agent, the principal shifts all of the risk to the agent by "selling" the uncertain output to the agent for a fixed payment. When there are $n \geq 2$ risk neutral agents bidding for the privilege of working for the principal, we show the existence of a Pareto optimal contract-auction scheme that maximizes the ex ante surplus and transfers all of this surplus to the principal.

I. INTRODUCTION

In the "standard" principal-agent model in which there is one principal contracting with one risk-neutral agent, the principal shifts all of the risk to the agent by "selling" the uncertain output to the agent for a fixed payment. This type of contract yields optimal risk sharing and ex ante Pareto efficiency. (See, for example Shavell [1979].) If, instead, there are $n \geq 2$ risk neutral agents bidding for the privilege of working for the principal, can we find an optimal risk-sharing contract that induces efficiency in the bidding process and that maximizes the principal's share ex ante? The purpose of this paper is the definition of just such a contract.

The scenario in which we will examine this problem is in the context of government contracting. As is pointed out by Riordan and Sappington [1987], government contracting (or the awarding of monopoly franchises in their paper) consists of three parts. The first is the selection of the contractor, the second is the determination of how much to produce, and the third is the division of the surplus. The authors look at all three of these issues in a model that contains only adverse selection: the government cannot observe the marginal costs of the potential franchisees ex ante. There is no moral hazard in this model and the only "uncertainties" arise because of the asymmetry of information between the bidders and the government. A bidding scheme is developed in which the most efficient bidder is revealed in equilibrium.

The model we propose is one that deals only with the selection of the contractor and the division of the surplus. We assume that the

government has already determined the level of production or the type of task to be performed, and needs only to select the most efficient contractor and divide the surplus. In a recent paper, McAfee and McMillan [1986] (hereafter referred to as M&M) model the bidding process for government contracts in the presence of both moral hazard and adverse selection. The government's objective, in M&M, is the selection of the most efficient contractor. There is adverse selection because the contractors' costs are not observable ex ante and moral hazard because the government cannot observe the cost reducing effort taken by the winning contractor. They examine the class of contracts that are linear in both the ex post cost and the bid price. Their "optimal" contract trades off between optimal risk sharing, the incentive to bid competitively, and the incentive to take action. With M&M's "optimal" contract, even when the agents are all risk neutral, the principal (government) is forced to bear some of the risk in order to stimulate bidding competition.

In this paper, we use the same basic model as M&M to demonstrate that it is not necessary to sacrifice optimal risk sharing. We demonstrate the existence of a Pareto optimal contract that results in the most efficient bidder being awarded the contract. The contract we propose is one in which the agents pay an "entrance fee" to bid for the contract; if the agent is the winning bidder the fee is refunded and the agent is paid the bid price and bears all of the risk. With this contract the total ex ante surplus available to divide between the principal and the bidding agents is maximized, and the principal

receives all of the surplus. Thus, our Pareto optimal contract-auction scheme strictly dominates the contract proposed by M&M.

The basic model is outlined in Section II, which also contains the statement of our results and their proofs. We provide a closed-form example in Section III. Conclusions are contained in the last section.

II. THE MODEL¹

The government (the principal) wishes to obtain bids for the performance of a particular task, where there are $n \geq 2$ potential contractors (agents) with the expertise to perform the task. Each agent has ex post costs to perform the task which depend on the agent's individual opportunity cost, a random cost factor that is common to all agents and the agent's cost reducing effort. It is assumed that ex post cost C_i is linear in these factors, so that it can be written as:

$$C_i \equiv C_i^* + W - \xi_i \quad (1)$$

where C_i^* = agent i 's opportunity cost

W = a random variable effecting costs, common to all agents;

$$E[W] = 0$$

ξ_i = agent i 's cost reducing effort

The government cannot observe C_i^* either ex post or ex ante. The agents each know their own C_i^* 's, but not those of the other agents. The distribution of W is common knowledge to the principal and all of the agents; however, the realization of W is known only to the winning bidder. Thus, we have both a moral hazard problem, since ξ_i is unobservable and an adverse selection problem since it may not be in the agents' best interests to correctly reveal C_i^* .

When the government announces its intention to solicit bids, each of the potential bidders do not know their own individual C_i^* 's. These values are revealed to them only after they have an opportunity to look

at the government's specifications for the task it wants performed. For example, the government may announce that it will solicit bids for a new fighter plane. The n potential bidders are those firms with the ability to design and build such planes. However, the firms will not know the value of C_i^* until after the firm has been able to see the exact features the government wishes the fighter to have.

Each one of the n potential bidders assumes that its own opportunity cost as well as the opportunity costs of the other bidders are independently generated by the same distribution. The only difference between the n agents is the opportunity cost C_i^* . We can think of these agents as having general expertise in the performance of a general class of tasks. However, each of the agents may have a comparative advantage for more precisely defined tasks within the general class. Thus, each of the agents have an opportunity cost that is an independent realization of the same distribution. We assume that the C_i^* are in $[C_l, C_h]$ and have a distribution function $G(a)$ such that²

$$P(C_i^* < a) = G(a). \quad (2)$$

The distribution function $G(\cdot)$ is common knowledge to all of the agents and the principal. Each agent, after learning his or her own value of C_i^* , believes that the C_j^* for the $(n-1)$ remaining agents are all independent random variables generated by the distribution in (2).

Since we assume the agents to be identical, except for their individual opportunity cost C_i^* , the dollar costs to the agents of

providing cost-reducing effort is the same for all agents. We denote this cost by $h(\xi)$, where:

$$h' > 0 \text{ and } h'' > 0. \quad (3)$$

We are therefore assuming that the cost to the agent of reducing the cost of the project is increasing at an increasing rate. This function is assumed to be known by the principal.

The agent's wealth is a function of the payment received from the principal and the dollar costs of effort. The payment, in turn, depends on what the principal can observe: the agent's bid and the ex post cost. If X is the payment, we assume that the risk neutral agents have utility that is separable in payment and cost of effort:

$$X - h(\xi_i). \quad (4)$$

The government wants to design a contract-auction scheme that is Pareto optimal. The specification of the scheme includes the definition of (i) how the winning contractor is chosen (the auction) and (ii) how the winning and losing bidders are rewarded (the contract). Since the winner's and losers' rewards are merely transfer payments, the total ex ante surplus is just

$$E[-(C^*+W-\xi) - h(\xi)] \quad (5)$$

which is the negative of the total costs of the project. The costs are made up of the direct costs of the project and the agent's costs of effort. A Pareto optimal contract-auction scheme would maximize

the ex ante expected surplus and give all of the surplus to the government.

In the next section we define such a Pareto optimal contract-auction scheme and demonstrate its optimality.

III. OPTIMAL RISK SHARING CONTRACTS

In McAfee and McMillan, linear contracts of the following form are analyzed:³

$$\alpha C_i + (1-\alpha)b_i, \quad (6)$$

where b_i is the bid price and C_i is ex post cost. If $\alpha = 0$, then we have a fixed-price contract; if $\alpha = 1$, we have a cost plus contract; and if $0 < \alpha < 1$, we have an incentive contract. Losing bidders receive their reservation wage and the winning agent is chosen with a first-price sealed bid auction in which minimum bid wins. When agents are risk neutral and $n = 1$, the optimal risk sharing contract is one in which $\alpha = 0$; that is, a contract in which the agent receives a fixed price and bears all of the risk. McAfee and McMillan demonstrate that for the class of contracts defined in (6), α never equals zero for any finite number of bidders (i.e., for all $n \geq 2$).

The Pareto optimal contract-auction scheme we define is one in which (i) the bidders pay an "entrance fee" to submit a bid, which is refunded to the winning bidder; (ii) the winning bidder is chosen by a first (lowest) price sealed bid auction; and (iii) the winning bidder receives the bid price and bears all of the ex post costs.

To demonstrate the optimality of this scheme, we must demonstrate that the total ex ante surplus is maximized and can be transferred to the principal. The agents' bids will be denoted by b_i and the entrance fee by F . The chronology of the process is as follows: the agents pay the fee F , they then submit a bid, and the winning bidder chooses the level of cost reducing effort, ξ_i , to provide.

If agent i is the winning bidder, and since the agent is risk neutral, effort will be chosen to solve:

$$\text{MAX}_{\xi_i} E_W[b_i - C_i] - h(\xi_i). \quad (7)$$

Note that the entry fee F does not enter into this computation since we are assuming that the decision about how much effort to provide is made after the bidding process. Since $E_W[W] = 0$, we can rewrite (6) as:

$$\text{MAX}_{\xi_i} b_i - C_i^* + \xi_i - h(\xi_i). \quad (8)$$

The agent thus chooses effort to maximize $\xi_i - h(\xi_i)$. By assumption on $h(\cdot)$, this maximizing value is unique and is independent of i . Denoting the maximizing value by ξ_i^* , the expected utility of the winning bidder can then be written as:

$$b_i - C_i^* + \xi_i^* - h(\xi_i^*) \quad (9)$$

Since the winning bidder is chosen by a first-price, sealed bid auction, a symmetric Nash equilibrium is characterized by a bidding strategy $B(C_i^*) = b_i$ that maximizes the agent's expected utility. The probability, in equilibrium, that agent i is the winner is just the probability that $B(C_i^*) < b_j$ for all $j \neq i$. Denote this probability by $H(B^{-1}(b_i))$:

$$H(B^{-1}(b_i)) \equiv [1 - G(B^{-1}(b_i))]^{n-1}. \quad (10)$$

Note that (10) is a consequence of the assumption that the C_j^* are independent draws from the same distribution.

The agent's bid must then solve:

$$\text{MAX}_{b_i} H(B^{-1}(b_i)) [b_i - C_i^* + \xi_i^* - h(\xi_i^*)] + (1 - H(B^{-1}(b_i))) [-F]. \quad (11)$$

Once the agent has decided to submit a bid and has paid the entrance fee F , the agent looks at the contract requirements and thus learns his or her own opportunity cost C_i^* . The optimal bid is therefore chosen after learning C_i^* and the expression in (11) is the expected utility of the agent given that they have decided to bid. Under our assumptions, it is again straightforward to verify that the equilibrium bidding strategy $B^*(C_i^*)$ satisfies the following first order differential equation:

$$B'(C_i^*; F) = \frac{(n-1)g(C_i^*)}{1-G(C_i^*)} (B(C_i^*; F) - C_i^* + \xi_i^* - h(\xi_i^*) + F) \quad (12)$$

Since $B(C_i^*; F) - C_i^* + \xi_i^* - h(\xi_i^*)$ is the expected utility if the agent wins the contract, it must be greater than zero in equilibrium. Therefore, $B'(C_i^*; F) > 0$ and the bid is increasing in cost. Using a first-price auction, the lowest bidder is the agent with the lowest cost and the contract is awarded to the least cost agent.

The ex ante expected surplus is just:

$$\begin{aligned} E[-C_i^* + W + \xi_i^* - h(\xi_i^*)] = \\ -E[C_i^*] + \xi_i^* - h(\xi_i^*) \end{aligned} \quad (13)$$

since $E[W] = 0$. Given that the equilibrium bidding strategy and a first-price auction results in the least cost bidder being selected, $-E[C_i^*]$ is maximized in our scheme.

We now must demonstrate that $\xi_i - h(\xi_i)$ is maximized as well. This result follows immediately from the definition of ξ_i^* :

$$\xi_i^* = \arg \max b_i - C_i^* + \xi_i - h(\xi_i). \quad (14)$$

The only thing that remains to be shown is the existence of an entrance fee F that transfers all of the surplus to the principal ex ante.

The expected utility of an agent after C^* is realized, but before bidding, is:

$$H(C^*)[B^*(C^*;F) - C^* + \xi^* - h(\xi^*)] + (1-H(C^*))[-F]. \quad (15)$$

The ex ante expected utility of the agent can then be written as:

$$\int g(C^*)H(C^*)[B^*(C^*;F) - C^* + \xi^* - h(\xi^*) + F]dC^* - F. \quad (16)$$

To show the existence of an F that makes this expected utility equal to zero, we first show that⁴

$$B(C^*;F) = B(C^*;0) - F. \quad (17)$$

Recall that

$$B^{*'}(C^*;F) = \frac{(n-1)g(C^*)}{1-G(C^*)} (B^*(C^*;F) - C^* + \xi^* - h(\xi^*) + F)$$

By definition, $B^*(C^*;0)$ solves this equation when $F = 0$. Let

$B^*(C^*;F) = B^*(C^*;0) - F$. Then,

$$\begin{aligned} B^{*'}(C^*;0) &= \frac{(n-1)g(C^*)}{1-G(C^*)} (B^*(C^*;0) - C^* + \xi^* - h(\xi^*)) \\ &= \frac{(n-1)g(C^*)}{1-G(C^*)} (B^*(C^*;0) - C^* + \xi^* - h(\xi^*)), \end{aligned}$$

which is true from the definition of $B^*(C^*;0)$. Substituting $B^*(C^*;0) - F$ into the agents expected utility gives us:

$$\int g(C^*)H(C^*)[B^*(C^*;0) - C^* + \xi^* - h(\xi^*)]dC^* - F. \quad (18)$$

Therefore,

$$F^* = \int g(C^*)H(C^*)[B^*(C^*;0) - C^* + \xi^* - h(\xi^*)]dC^*.$$

We can now state our main result, whose proof is immediate from the preceding discussion.

THEOREM

The ex ante Pareto optimal contract-auction scheme is one in which (i) the agents pay an entrance fee to submit a bid, which is refunded to the winning bidder; (ii) the contract is awarded to the lowest bidder with a sealed-bid auction; and (iii) the winning bidder receives the bid price and bears all of the ex post cost of the project.

It should be noted that the fixed entrance fee F^* is the agents' ex ante expected utility in equilibrium when no entrance fee is charged. This is analogous to the case when there is only one risk neutral

agent; there, the agent is charged a fixed payment that equals the surplus that the agent would have captured if no payment was made.

McAfee and McMillan's "optimal" contract in which the principal bears some of the risk associated with the ex post cost is strictly dominated by our contract. The amount of information necessary to implement our scheme is no greater than the amount of information necessary to implement their scheme; in the next section we use their example to obtain closed form solutions and demonstrate that our scheme actually requires less information (in this special case). In addition, the Pareto optimal scheme does not require ex post observability of the actual costs of the project, while the scheme proposed by McAfee and McMillan does require such verification.

III. AN EXAMPLE

When the C_i^* are i.i.d. exponential with lower support C_ℓ and parameter μ , and $h(\xi)$ is quadratic with $h'' = h_0$, we can compute the entrance fee F^* , the equilibrium bid $B^*(C_i^*; F^*)$ and the surplus.

Using the definition of ξ^* and equations (12) and (17), we have

$$B^*(C_i^*; F^*) = B^*(C_i^*; 0) - F^* = \frac{1}{\mu(n-1)} - \frac{1}{2h_0} + C_i^* - F^*.$$

Substituting into equation (18) and noting the $B^*(C_i^*; 0) - C_i^*$ is independent of C_i^* , we have

$$F^* = \left[\frac{1}{\mu(n-1)} - \frac{1}{2h_0} + \frac{1}{2h_0} \right] \frac{1}{n} = \frac{1}{\mu(n-1)n}$$

since $\int g(C^*)H(C^*)dC = \frac{1}{n}$.

The equilibrium bid is then

$$B^*(C_i^*; F^*) = \frac{1}{\mu(n-1)} - \frac{1}{2h_0} + C_i^* - \frac{1}{\mu n(n-1)} = \frac{1}{\mu n} - \frac{1}{2h_0} + C_i^*.$$

Ex ante expected surplus can be computed as

$$-n \int g(C^*)H(C^*)C^* + \xi^* - h(\xi^*) = -n \int g(C^*)H(C^*)C^* + \frac{1}{2h_0}.$$

For the exponential distribution, the expected value of the minimum of n i.i.d. random variables is

$$C_\ell + \frac{1}{\mu n}$$

which gives an ex ante expected surplus of

$$- C_\ell - \frac{1}{\mu n} + \frac{1}{2h_0}.$$

The nice feature of the exponential distribution is that the government need only know the number of bidders n and the standard deviation of costs in order to implement the scheme. In the contract proposed by McAfee and McMillan, the "optimal" fraction of the cost to be shared by the principal equals $\frac{h_0}{\mu(n-1)}$. Thus, the government must know the disutility of effort of the agents, as well as the number of bidders and the standard deviation of costs, to implement the scheme for the functions discussed here. In general, however, F^* will depend on the agents' disutility of effort.

IV. SUMMARY

We have shown the existence of a Pareto optimal contract-auction scheme that leads the principal to maximize his or her surplus when there are n risk neutral bidders with unobservable costs bidding for a contract. A first-price sealed bid auction is used to select the winning bidder. In equilibrium, the lowest cost bidder is revealed. The contract charges the bidders an "entrance" fee to bid; the winning (lowest) bidder receives exactly the bid price and bears all of the risk.

This optimal risk sharing contract strictly dominates (for the principal) any incentive contract that reveals the lowest cost bidder in equilibrium. With the latter, it must be possible to observe actual costs ex post; with the optimal risk sharing contract, ex post verification is not necessary.

FOOTNOTES

¹The model we define here was motivated by the model developed in McAfee and McMillan [1986].

²Note that C_ℓ may equal zero and C_h may equal ∞ .

³McAfee and McMillan show that without loss of generality any contract that is linear in ex post cost and bid price can be expressed as in (5).

⁴The insight into this fact is due to Richard Englebrecht-Wiggans.

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